Estimation and Detection Based on Correlated Observation from a Heterogeneous Sensor Network

Sima Sobhiyeh[1] and Mort Naraghi-Pour[1].

1. Division of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA, 70803

ABSTRACT

This paper considers the problem of parameter estimation and hypothesis detection based on a network of heterogeneous sensors. The data is assumed to be correlated both among the samples collected over time and among the data collected by different sensors. We model the correlation in the data based on the copula theory and a Markov chain. The proposed algorithm uses the EM algorithm to estimate the unknown parameters of the model and detect the state of nature. Numerical results show significant improvement in detection and estimation performances when the correlation in the data is considered.

INTRODUCTION

In this paper we consider detection in a heterogeneous sensor network. Many approaches to data fusion assume that, given each hypothesis, the sensors' local measurements are conditionally independent. However, in most practical cases this assumption fails as the data collected by the sensors are correlated over time, as well as among the sensors. Recently, in [1],[2], the copula theory was used to model the correlation among sensors' decisions. But the observation samples of each sensor were assumed to be independent and identically distributed (iid) over time.

In this study, we assume that the data received from the sensors are correlated not only among the sensors, but also over time. We devise a model based on the copula theory and Markov chains to model the correlation in the. Next, we develop a method based on the expectation maximization (EM) algorithm to estimate the unknown parameters of the underlying joint and marginal PDFs, and to detect the hypotheses at each time.

PROBLEM FORMULATION

- There are L heterogeneous sensors.
- Each sensor collects data from nature at times t=1:T.
- Data are sent to the fusion center (FC) forming the matrix D.
- The sensors' data are assumed to be correlated.
- Given D, the FC estimates unknown parameters of the joint distribution of sensors' data and detects the state of nature.
- The state of nature at each time t is $H_i,\ i=0,1$.
- We define the matrix, $H \triangleq [h_{i,t}]$, $h_{i,t}=1$ if H_i holds at time t.
- If $h_{i,t}=h_{i,t-1}$, $d_{l,t}$ is correlated with $d_{l,t-1}$.
- If $h_{i,t}\neq h_{i,t-1}$, $d_{l,t}$ and $d_{l,t-1}$ are independent.

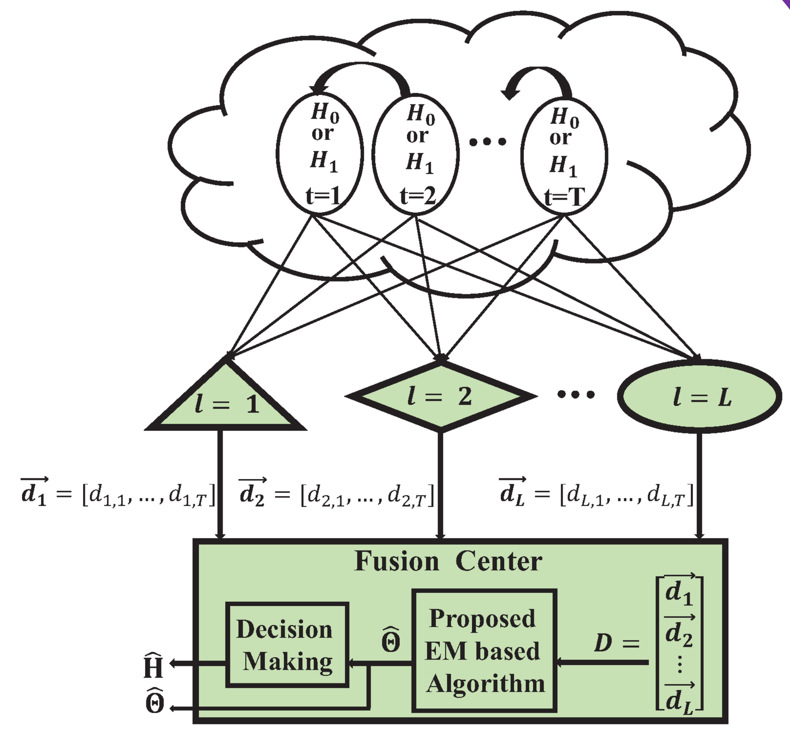

Fig. 1: Pictorial description of problem formulation

SYSTEM MODEL

Markov chain model ➡ modeling correlation over time when it exists.
Copula theory ➡ modeling the correlation among measurements of different sensors.

$P(D,H|\Theta)$

$$= \prod_{t=1}^{T}\prod_{i=0}^{1}\left[\frac{\tilde{\phi}_{i,1-i,t}}{\phi_{1-i,t}}\, c_1\left(F_i^{(1)}\left(d_{1,t};\psi_i^{(1)}\right),\dots,F_i^{(L)}\left(d_{L,t};\psi_i^{(L)}\right);\lambda_{1,i}\right)\prod_{l=1}^{L} f_i^{(l)}\left(d_{l,t};\psi_i^{(l)}\right)\right]^{h_{i,t}h_{1-i,t-1}}$$

$$\times\left[\frac{\tilde{\phi}_{i,i,t}}{\phi_{i,t}}\, c_1\left(F_i^{(1)}\left(d_{1,t}|d_{1,t-1};\tilde{\psi}_i^{(1)}\right),\dots,F_i^{(L)}\left(d_{L,t}|d_{L,t-1};\tilde{\psi}_i^{(L)}\right);\lambda_{2,i}\right)\prod_{l=1}^{L} f_i^{(l)}\left(d_{l,t}|d_{l,t-1};\tilde{\psi}_i^{(l)}\right)\right]^{h_{i,t}h_{i,t-1}}$$

- $\Theta=\{\tilde{\Phi},\Psi,\Lambda\}$: the set of unknown parameters, $\tilde{\Phi}=[\tilde{\phi}_{i,j,t}]$, $\Psi=\left[\psi_i^{(l)}\right]$, $\Lambda=\{\lambda_{1,i},\lambda_{2,i}\}$
- $\tilde{\phi}_{i,j,t}=\Pr\left(h_{i,t}=1,h_{j,t-1}=1\right)$, $j=i,1-i,\ i=0,1.$ $\quad \phi_{i,t}=\tilde{\phi}_{i,1-i,t}+\tilde{\phi}_{i,i,t}=\Pr\left(h_{i,t}=1\right)$.
- $F_i^{(l)}(x;\psi_i^{(l)})$: marginal distribution of the lth sensor's measurements under H_i with unknown parameter $\psi_i^{(l)}$.
- $c_j(\underline{x};\lambda_{j,i})$: copula density with unknown parameter $\lambda_{j,i}$, $j=1$ accounts for when the data samples are not correlated over time, $j=2$ accounts for when the data samples are correlated over time.

THE PROPOSED ALGORITHM

The EM algorithm is used to find $\hat{\Theta}$.

The E-step:

$Q(\Theta;\Theta^{old}) \triangleq E_{H|D;\Theta^{old}}[\log P(D,H|\Theta)]$

$$= \sum_{t=1}^{T}\sum_{i=0}^{1}\sum_{l=1}^{L}\alpha_1(i,t)\left[\frac{1}{L}\log\frac{\tilde{\phi}_{i,1-i,t}}{\phi_{1-i,t}}+\frac{1}{L}\log c_1\left(F_i^{(1)}\left(d_{1,t};\psi_i^{(1)}\right),\dots,F_i^{(L)}\left(d_{L,t};\psi_i^{(L)}\right);\lambda_{1,i}\right)+\log f_i^{(l)}\left(d_{l,t};\psi_i^{(l)}\right)\right]$$

$$+\alpha_2(i,t)\left[\frac{1}{L}\log\frac{\tilde{\phi}_{i,i,t}}{\phi_{i,t}}+\frac{1}{L}\log c_2\left(F_i^{(1)}\left(d_{1,t}|d_{1,t-1};\psi_i^{(1)}\right),\dots,F_i^{(L)}\left(d_{L,t}|d_{L,t-1};\psi_i^{(L)}\right);\lambda_{2,i}\right)+\log f_i^{(l)}\left(d_{l,t}|d_{l,t-1};\psi_i^{(l)}\right)\right]$$

Where $\alpha_1(i,t)\triangleq E_{H|D;\Theta^{old}}\left[h_{i,t}h_{1-i,t-1}\right]$, $\alpha_2(i,t)\triangleq E_{H|D;\Theta^{old}}\left[h_{i,t}h_{i,t-1}\right]$.

The M-step:

$\underset{\tilde{\phi}_{i,j,t}}{\operatorname{argmax}}\, Q(\Theta;\Theta^{old})$ subject to $\sum_{i=0}^{1}\tilde{\phi}_{i,1-i,t}+\tilde{\phi}_{i,i,t}=1$ ➡ $\begin{cases}\tilde{\phi}_{i,i,t}=\alpha_1(i,t)\\ \tilde{\phi}_{i,1-i,t}=\alpha_2(i,t)\end{cases}$

$\underset{\psi_i^{(l)}}{\operatorname{argmax}}\, Q(\Theta;\Theta^{old})$ subject to $\int_{-\infty}^{\infty} f_i^{(l)}\left(x;\psi_i^{(l)}\right)dx=1$

$\underset{\lambda_{j,i}}{\operatorname{argmax}}\, Q(\Theta;\Theta^{old})$ subject to $\int_0^1\dots\int_0^1 c_j(\underline{x};\lambda_{j,i})d\underline{x}=1$

Case Study: Gaussian Copula, first order Auto-Regressive (AR1) model

Let $f_i^{(l)}\left(d_{l,t};\psi_i^{(l)}\right)\sim\mathcal{N}\left(\psi_i^{(l)},(\sigma_i^{(l)})^2\right)$, $d_{l,t}=\eta d_{l,t-1}+v_{i,t}^{(l)}$, $\left\{v_{i,t}^{(l)}\right\}$ i.i.d Gaussian innovation process with mean $\psi_i^{(l)}$ and standard deviation $\sigma_i^{(l)}$.

$$\underline{\boldsymbol{\psi}_i}=\left(\sum_{t=1}^{T}\sum_{j=1}^{2}\alpha_j(i,t)\,\lambda_{j,i}^{-1}\right)^{-1}\left(\sum_{t=1}^{T}\alpha_1(i,t)\,\lambda_{1,i}^{-1}\underline{\boldsymbol{d}}_t+\alpha_2(i,t)\lambda_{2,i}^{-1}(\underline{\boldsymbol{d}}_t-\eta\underline{\boldsymbol{d}}_{t-1})\right),\ \underline{\tilde{\boldsymbol{\psi}}_i}=\eta\underline{\boldsymbol{d}}_{t-1}+\underline{\boldsymbol{\psi}_i}$$

$$\lambda_{j,i}=\frac{\sum_{t=1}^{T}\alpha_j(i,t)\underline{\boldsymbol{y}}_{j,i}(t)\underline{\boldsymbol{y}}_{j,i}(t)^{Tr}}{\sum_{t=1}^{T}\alpha_j(i,t)}$$

Where $\underline{\boldsymbol{\psi}_i}=\left[\psi_i^{(k)}\right]_{L\times1}$, $\underline{\boldsymbol{d}}_t=[d_{k,t}]_{L\times1}$, $\underline{\boldsymbol{y}}_{j,i}(t)=\left[y_{j,i}^{(k)}(t)\right]_{L\times1}$, $y_{1,i}^{(k)}(t)=\frac{d_{k,t}-\psi_i^{(k)}}{\sigma_i^{(k)}}$, $y_{2,i}^{(k)}(t)=\frac{d_{k,t}-\tilde{\psi}_i^{(k)}}{\sigma_i^{(k)}}$, and Tr denotes matrix transpose.

SIMULATION AND RESULTS

Initialization: $\tilde{\phi}_{i,j,t}=.25$, $\lambda_{j,i}=I_L$, and the initial values of $\psi_i^{(l)}$ are obtained from the unsupervised method of K-means.

Convergence: in less than 5 iterations.

Comparison:

- Case 1: Proposed model, considering the dependence both among the sensors and over time samples.
- Case 2: Dependence in the data collected by different sensors is modeled but dependence in the samples collected over time is ignored.
- Case 3: Dependence in the data collected by different sensors is ignored but dependence in the samples collected over time is modeled.
- Case 4: From right to left, $\Delta_H,\Delta_\Lambda,\Delta_\Psi$, and execution time Dependence both among the sensors and over time samples are ignored.

Evaluation Criterion: detection error: $\Delta_H\triangleq\frac{1}{2T}\sum_{i=0}^{1}\sum_{t=1}^{T}\left|h_{i,t}-\hat{h}_{i,t}\right|$, estimation error: $\Delta_\Psi\triangleq\frac{1}{2L}\sum_{i=0}^{1}\sum_{l=1}^{L}\frac{\left|\psi_i^{(l)}-\hat{\psi}_i^{(l)}\right|}{\psi_i^{(l)}}$, $\Delta_\Lambda\triangleq\frac{1}{4L^2}\sum_{j=1}^{2}\sum_{i=0}^{1}\sum_{n=1}^{L^2}\frac{|\lambda_{j,i}(n)-\hat{\lambda}_{j,i}(n)|}{\lambda_{j,i}(n)}$

Discussion:

- Case 1 has significantly lower detection and parameter estimation error and this improvement increases as the number of sensors increase.
- As T increases, estimation error decreases but since the number of hypotheses to be detected is T, the detection error reaches a floor.
- For a larger L, a larger T is required to achieve the best possible detection performance that the proposed algorithm can offer.
- The performance of Case 2 (blue curves) is worse than the performance of Case 4 (red curves). Thus, if the data are correlated over time and among the sensors (as is in many practical applications), then ignoring the dependence over time and only modeling the dependence among the sensors, results in a worse performance than ignoring the dependencies all together.

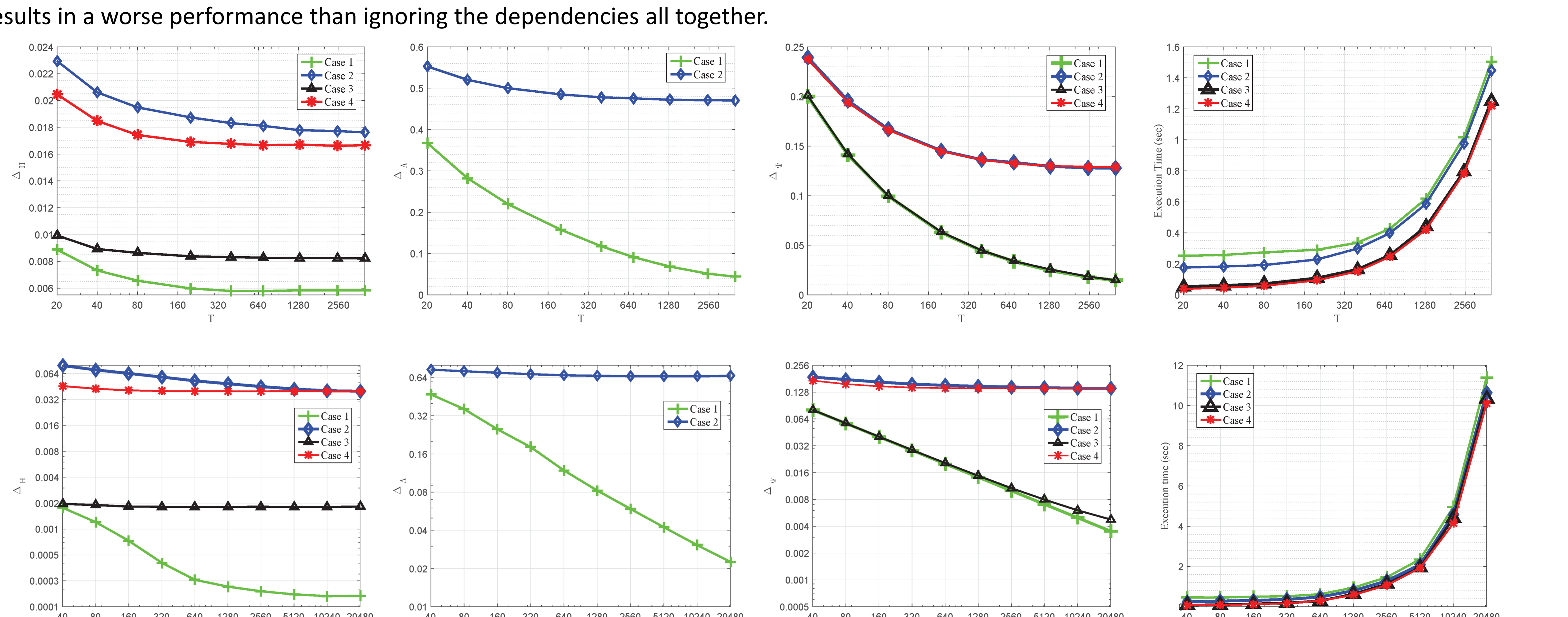

Fig. 2: Estimation and detection performances; from left to right, $\Delta_H,\Delta_\Lambda,\Delta_\Psi$, and execution time versus T for Case 1-4. Top row: four sensors, L=4. Bottom row: eight sensors, L=8.

CONCLUSION

- We consider the problem of binary hypothesis testing in a sensor network consisting of heterogeneous sensors collecting correlated data. The data is assumed to be correlated over time as well as among the sensors. Moreover, it is assumed that the complete set of statistical parameters of the data collected by the sensors is not available.
- We propose a method based on the expectation maximization (EM) algorithm to estimate the unknown parameters and to detect the state of nature.
- An illustrative example is presented with the Gaussian copula and where a first order auto-regressive model is employed to establish the dependence over time samples.
- Numerical results show that when the dependence among the sensors' data and among the samples from each sensor are employed in the model, a significantly better performance in terms of hypothesis detection as well as model parameter estimation is achieved.

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