Optimal PMU Placement using Sensitivity Analysis for Power Systems Fault Location

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INTRODUCTION

- **This research presents a novel algorithm to find optimal sets of Phasor Measurement Units (PMUs) in power systems using measurement sensitivity analysis aiming for fault detection without multi-estimation.**
- **Synchronized Phasor Measurement Units (PMUs) roles in power systems operation, control, and protection are prominent and constantly developing.**
- **The proposed algorithm generalizes the impedance method in fault detection through optimizing PMU utilization in order to detect a fault with desired precision in interconnected power systems.**
- **A normal observable system is when voltage phasors for all the system buses are available, while fault observable is a system during fault where voltage phasors of all buses and current phasors at any end of all lines are determinable Normal observability in power system is defined as knowing the voltage phasors of all the buses .**
- **While many PMU placement approaches are proposed to solve normal observability (under normal operating condition) for power systems, there are a very limited number of related studies that target fault observability.**
- **Though the available approaches take advantage of various algorithms to impose observability constraints, the important issue of measurement precision and its impact on OPP and fault location is considered in very few literatures.**

Methodology

❖ *Problem Statement*

- ✓ **When a fault occurs in the system, according to the location and impedance of the fault all voltages and currents of the network change including at PMU location buses.**
- ✓ **the problem rises when more than one fault (with possibly different impedances and locations) cause the same change in the voltages and currents at PMU busses which is called multi estimation.**
- ✓ **What is the PMUs optimal location set to avoid multi estimation, and to use measured voltage and current changes at the PMU locations to uniquely identify a fault, i.e. resulting in a fault observable system?**
- ✓ **Develop a fault locator by utilizing obtained optimal PMU sets via artificial neural networks (ANNs).**

❖ *Definitions*

Fault: A fault is identified by value $F = (l_f, D, R_f)$ where $l_f \in L_f = \{1,2,\dots,L\}$ is the line number where fault occurs with L being the total number of lines in the power system, $D \in \mathrm{D} = [0,1]$ is the normalized distance of the fault with respect to one of the line end buses ($D = \frac{\text{length(lp)}}{\text{length(lk)}}$ from Fig. 1), and $R_f \in \mathrm{R}_f = [0, R_{max}]$ is the fault resistance.

Observant bus: Bus $h \in \{1,2,\dots,N\}$, with N being the total number of power system buses, where a measurement device capable of measuring the bus voltage and currents (of the lines connected to that bus) is installed, is an observant bus.

Observant set: A set $H \subseteq \{1,2,\dots,N\}$ of observant buses is called an observant set.

Multi-estimation: Multi-estimation is a condition where different faults cause similar measured values in an observant set.

❖ *Power System Measurement Sensitivity Analysis*

The approach presented in this paper is built upon the classical fault analysis.

Figure 1.a illustrates the unfaulty network with known impedance matrix Z_0, voltages, and currents. Also, the figure depicts the four steps which are developed to modify Z_0 and obtain Z_4. Explicit form of Z_4 is needed to develop the indices as explained next, similarly for Y_{bus}.

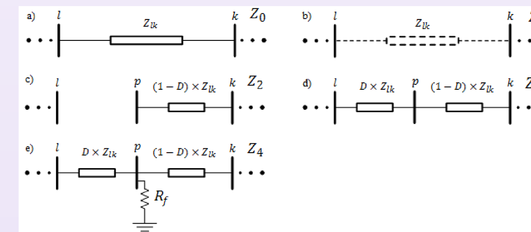
Fig. 1 Steps for Z_{bus} modification: Z_0 through Z_3 are the steps of change in Z_{bus}

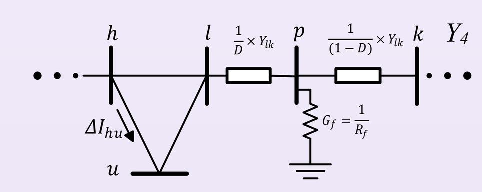
Fig. 2. Observant and adjacent buses in faulty system

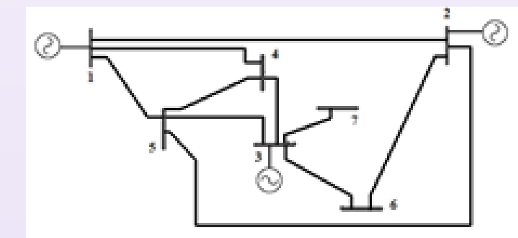
Fig. 3 IEEE 7-bus system

By using the standard fault analysis, the voltage changes at observant bus h, (when fault F occurs at bus p) can be described as

$$\Delta V_{h,F} = \frac{Z_3(h,p)}{Z_3(p,p)+R_f} \times V_{pref} \qquad (1)$$

where $Z_3(h,p)$ is the (h,p) entree of Z_3, $Z_3(p,p)$ is the system Thevenin impedance seen from imaginary bus p, and V_{pref} is the prefault voltage at the point of fault in the system. With the assumption of linear voltage drop along the transmission lines between buses and by ignoring line capacitances to avoid complexity, V_{pref} can be calculated as:

$$V_{pref} = V_l + (1-D) \times (V_l - V_k). \qquad (2)$$

A. Voltage Sensitivity Indices

If deviation from the voltage's normal value at observant bus h due to fault $F = (l_f, D, R_f)$ is represented by $\Delta V_{h,F}$, then the voltage sensitivity indices are defined as derivatives of fault distance D and impedance R_f with respect to $\Delta V_{h,F}$ as

$$S^{DV}_{h,F} = \frac{\partial D}{\partial \Delta V_{h,F}} \text{ and } S^{R_f V}_{h,F} = \frac{\partial R_f}{\partial \Delta V_{h,F}}, \qquad (3)$$

respectively.

B. Current Sensitivity Indices

An installed PMU on any grid bus measures the phasor of the bus voltage as well as those of the currents of all the connected lines.

In a similar manner to voltage sensitivity indices, current sensitivity indices can be defined for each line connected between observant bus h and an adjacent bus u. These indices are defined as

$$S^{DI}_{hu,F} = \frac{\partial D}{\partial \Delta I_{hu,F}} \text{ and } S^{R_f I}_{hu,F} = \frac{\partial R_f}{\partial \Delta I_{hu,F}}. \qquad (4)$$

The number of current sensitivity indices derived for each observant bus h in (4) is equal to the number of lines connected to bus h. Since $\Delta V_{h,F}$ and $\Delta V_{u,F}$ are available for any observant bus h within the network, line current changes can be expressed as

$$\Delta I_{hu,F} = \frac{\Delta V_{h,F} - \Delta V_{u,F}}{Z_{hu}} = -Y_4(h,u) \times (\Delta V_{u,F} - \Delta V_{h,F}) \qquad (5)$$

where Z_{hu} is the impedance of line hu, $Z_{hu} = (-Y_4(h,u))^{-1}$, and $Y_4(h,u)$ is the (h,u) entree of the admittance matrix that corresponds to Z_4 according to Figs. 1 and 2.

C. Sensitivity Requirements, an example on IEEE 7-Bus

In the proposed methodology, fault is analyzed by considering TP^D as "*target precision for D*" and TP^{R_f} as "*target precision for R_f*." Fault location range is $0 \le D \le 1$ on a power line and, thus, for a given $TP^D \le 1$, fault can be detected to be on one of $\frac{1}{TP^D}$ equally-spaced points on any power line, similarly for R_f. Subsequently, the desired upper limits for sensitivity indices introduced in (3) and (4) can be calculated as

$$S^{DV}_{h,F} \le \frac{TP^D}{TVE^V} = \varepsilon_{DV},\; S^{R_f V}_{h,F} \le \frac{TP^{R_f}}{TVE^V} = \varepsilon_{R_f V},\; S^{DI}_{hu,F} \le \frac{TP^D}{TVE^I} = \varepsilon_{DI}, \text{ and } S^{R_f I}_{hu,F} \le \frac{TP^{R_f}}{TVE^I} = \varepsilon_{R_f I} \qquad (6)$$

where TVE^V and TVE^I are total vector errors in voltage and current measurements, respectively. In this section $TP^D = 0.01$ and $TP^{R_f} = 0.05$ are the desired resolutions in the fault detection algorithm. That is, the final fault location using resulted PMU sets from the proposed algorithm should not have errors more than 1% and 5% from fault's actual D and R_f. Therefore, one has sensitivity requirements $\varepsilon_{DV} = 10$, $\varepsilon_{R_f V} = 50$, $\varepsilon_{DI} = 10$, and $\varepsilon_{R_f I} = 50$. Sensitivity requirement $S^{DV}_{h,F} \le \varepsilon_{DV} = 10$ for Bus 4 and all system is illustrated in Fig. 4 and 5, respectively.

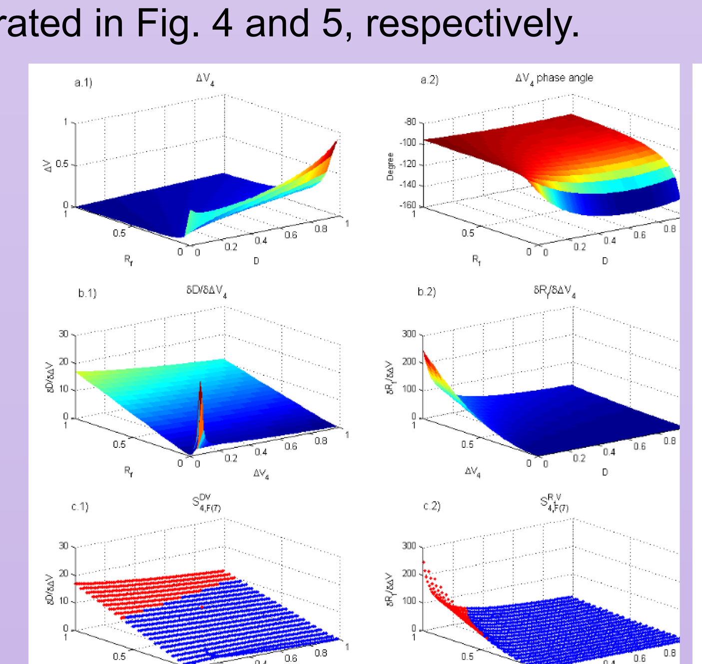
Figure 4. Bus 4 voltage sensitivities for faults on line between buses 4 and 1, F= $(7, 0 \le D \le 1, 0 \le R_f \le 1)$

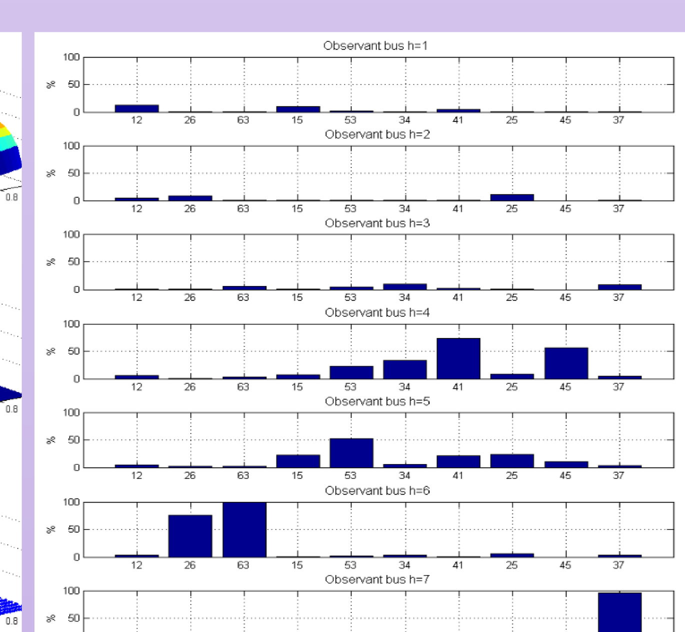
Figure 5. Percentages of satisfactory D-voltage sensitivity indices for all faulty lines per each observing bus regarding $S^{DV}_{h,F} \le \varepsilon_{DV} = 10$

D. Fault locator with unique function mapping using ANN

Assume H is OPP solution for IEEE 7-bus test system is the observant set H = {2,3}. Resulting set of measurements are:

$$M_{HF} = \{\Delta V_{h,F}, \Delta I_{hu,F} \mid h \in H, u \in U_h\} = \{V_2, I_{2g}, I_{21}, I_{26}, I_{25}, V_3, I_{3g}, I_{34}, I_{35}, I_{36}, I_{37}\}$$

Since our OPP solution prevents multi-estimation, there must exist a unique function mapping between M_{HF} and all possible $F = (l_f, D, R_f)$. This is pictured in Fig. 6. Also, the utilized network of ANNs is illustrated in Fig. 7.

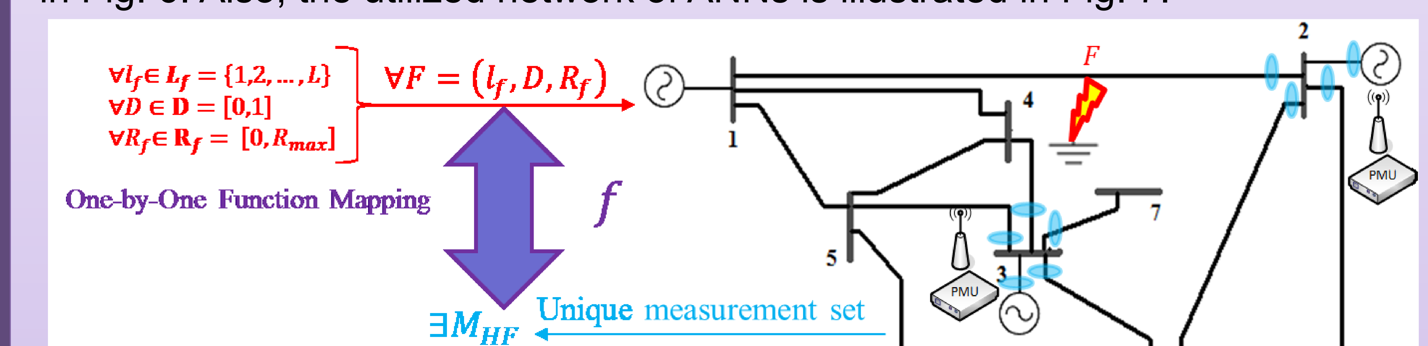

Fig 6. Unique function mapping using ANN

$$f_{(l_f, D, R_f)} = M_{HF} = (V_2, I_{2g}, I_{21}, I_{26}, I_{25}, V_3, I_{3g}, I_{34}, I_{35}, I_{36}, I_{37})$$

$$f^{-1}_{(V_2, I_{2g}, I_{21}, I_{26}, I_{25}, V_3, I_{3g}, I_{34}, I_{35}, I_{36}, I_{37})} = F = (l_f, D, R_f)$$

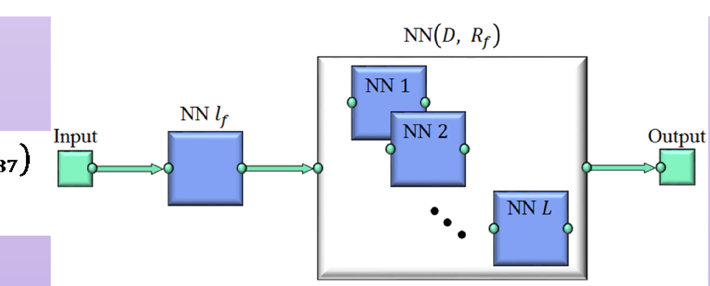

Fig. 7. Artificial Neural Networks structure

RESULTS

- **The IEEE 7-bus, 14-bus and 30-bus benchmarks are used as the case studies.**
- **Various Target Precisions and measurement accuracies are considered and results are provided for IEEE 7-bus in Table I.**
- **Bold OPP sets are used for ANN training and fault locator application. The percentage of correct fault detection is also provided in results.**

TABLE I IEEE 7-bus OPP and ANN results for various target precisions and different measurements accuracies

IEEE 7-bus OPP (R_f max 0.1 pu)				ANN		
				Percentage estimation accuracy		
TVE^V	TVE^I	# PMUs	Optimal observant sets (PMU locations)	l_f	D (ave) (min)	R_f (ave) (min)
$TP^D = 0.01, TP^{R_f} = 0.01$, Total generated faults: 11000						
10^{-2}	10^{-2}	2	(1,2)-**(2,3)**	99.6	99.9 99.1	99.9 99.5
10^{-3}	10^{-2}	2	(1,2)-**(2,3)**	99.8	100 100	100 100
10^{-3}	10^{-3}	1	(1)-(2)-(3)-**(5)**	99.9	100 100	100 100
$TP^D = 0.05, TP^{R_f} = 0.05$, Total generated faults: 600						
10^{-2}	10^{-2}	1	(3)-**(5)**	99.1	99.1 91.6	100 100
10^{-3}	10^{-2}	1	(3)-**(5)**	99.1	99.1 91.6	100 100
10^{-3}	10^{-3}	1	(1)-(2)-(3)-(4)-**(5)**-(6)	100	99.1 91.6	100 100

CONCLUSIONS

- **A new sensitivity analyses is derive and corresponding indices are provided**
- **The required number of the PMUs for fault observability is significantly reduced compared to the past works**
- **The proposed algorithm is capable to reach a fault detection target precision for D and R_f when considering available voltage and current measurement accuracies**
- **ANN is utilized to estimate the unique function mapping available between OPP set measurements and fault criteria (l_f, D, R_f)**
- **High percentage of correct fault detection is yielded in random fault data sets which were not used in ANN training process**