MIMO Detection With Imperfect Channel State Information Using Expectation Propagation

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Introduction

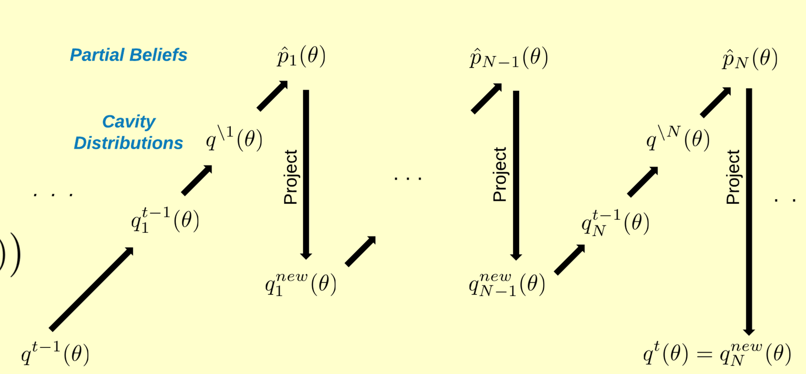
Symbol detection is challenging in massive MIMO systems:

- Optimal symbol detection in MIMO is an NP-hard problem.
  For an $M \times K$ MIMO system (M receiving and K transmitting antennas) which uses $\mathcal{M}$-ary modulation scheme, the optimal coherent receiver detects the transmitted symbols by solving $\hat{\mathbf{x}}_{ML} = \arg\max_{\mathbf{x}\in\mathcal{A}_{\mathcal{M}}^{K}} p(\mathbf{y}|\mathbf{x})$. This needs searching among $\mathcal{M}^K$ vectors.
- Under favorable propagation conditions, the channels of users are mutually orthogonal: linear detectors, such as ZF and MMSE, will have descent performances. However, the channel orthogonality is not always guaranteed in practice ($K \ll M$ reduces the overal system's capacity and in some environments increasing M does not leads to channel orthogonality).
- We need nonlinear detection algorithms to achieve better performance.
  – Cost: higher complexity.
  – Examples: BP [1], GTA [2], GTA-SIC [3], EP [4].
- the detection performances highly depend on the quality of Channel State Information (CSI).

Expectation Propagation (EP) [5]

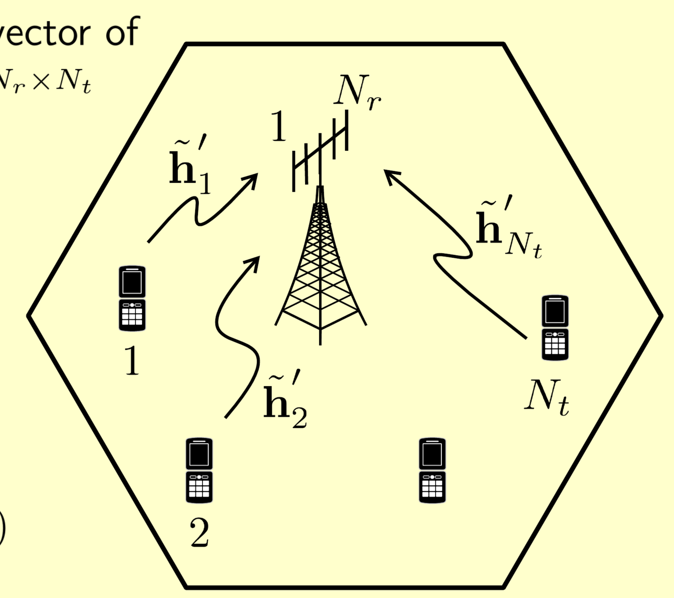
- Assume θ as the vector of unkown variables, and $p(\theta)$ as the joint a posteriori distribution of unknowns.
- The statistical inference from $p(\theta)$ is generally very complex. Finding the marginals needs extensive multi-dimensional integrations.
- This is not the case if $p(\theta)$ belongs to $\mathcal{F}$, the exponential family of distributions.
- **EP algorithm**: Iteratively approximating a factorized distribution, such as $p(\theta) = \prod_{n=1}^{N} p_n(\theta)$, by a member of $\mathcal{F}$, such as $\underbrace{q(\theta)}_{\in\mathcal{F}} \propto \prod_{n=1}^{N} \underbrace{q_n(\theta)}_{\in\mathcal{F}}$.

Refining the n-th factor:
1- Calculating the cavity PDF
$q^{\backslash n}(\theta) \propto q(\theta)/q_n(\theta)$
2- Calculate the partial PDF
$\hat{p}_n(\theta) = q^{\backslash n}(\theta)p_n(\theta)/Z_n$
3- Update $q(\theta)$
$q_n^{new}(\theta) = \arg\min_{f\in\mathcal{F}} KL(\hat{p}_n(\theta)\|f(\theta))$
4- Update $q_n(\theta)$
$q_n(\theta) \propto q_n^{new}(\theta)/q^{\backslash n}(\theta)$



System Model

- Reverse link of a MIMO system
- N_r receiving antennas, N_t single-antenna terminals ($N_r \geq N_t$)
- $\mathbf{h}_k^{'} \in \mathbb{C}^{N_r\times 1}$: **Rayleigh flat fading** channel vector of the k-th terminal, $\tilde{H}' = [\mathbf{h}_1^{'}\, \mathbf{h}_2^{'} \ldots \mathbf{h}_{N_t}^{'}] \in \mathbb{C}^{N_r\times N_t}$
  $\mathbf{h}' = \text{vec}(\tilde{H}') \sim \mathcal{CN}(\mathbf{h}'|\mathbf{0}, \tilde{R}_h)$
- Transmitted symbols at one channel use:
  $\tilde{\mathbf{u}} = [\tilde{u}_1, \ldots, \tilde{u}_{N_t}]^T \in \tilde{\mathcal{A}}_M^{N_t\times 1}$
  $\tilde{\mathcal{A}}_M$: M-ary modulation constellation with average energy E_s
- The received vector: $\tilde{\mathbf{y}} = \tilde{H}'\tilde{\mathbf{u}} + \tilde{\mathbf{n}}$
  $\tilde{\mathbf{n}}$: the noise vector with $\tilde{\mathbf{n}} \sim \mathcal{CN}(\tilde{\mathbf{n}}|\mathbf{0}, \sigma_n^2 I_{N_r})$



EP Detection with Perfect CSI [4]

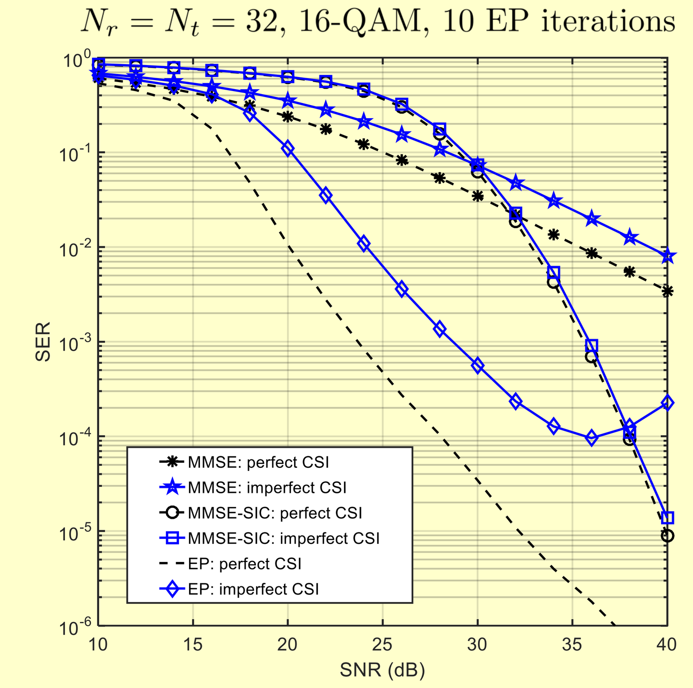
The received vector: $\tilde{\mathbf{y}} = \tilde{H}'\tilde{\mathbf{u}} + \tilde{\mathbf{n}}$

$$\underbrace{\begin{bmatrix}\Re(\tilde{\mathbf{y}})\\ \Im(\tilde{\mathbf{y}})\end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix}\Re(\tilde{H}') & -\Im(\tilde{H}')\\ \Im(\tilde{H}') & \Re(\tilde{H}')\end{bmatrix}}_{H}\underbrace{\begin{bmatrix}\Re(\tilde{\mathbf{u}})\\ \Im(\tilde{\mathbf{u}})\end{bmatrix}}_{\mathbf{u}} + \underbrace{\begin{bmatrix}\Re(\tilde{\mathbf{n}})\\ \Im(\tilde{\mathbf{n}})\end{bmatrix}}_{\mathbf{n}}, \quad \mathbf{n} \sim \mathcal{N}(\mathbf{n}|\mathbf{0}, \underbrace{\tfrac{1}{2}\sigma_n^2 I_{2N_r}}_{R_n})$$

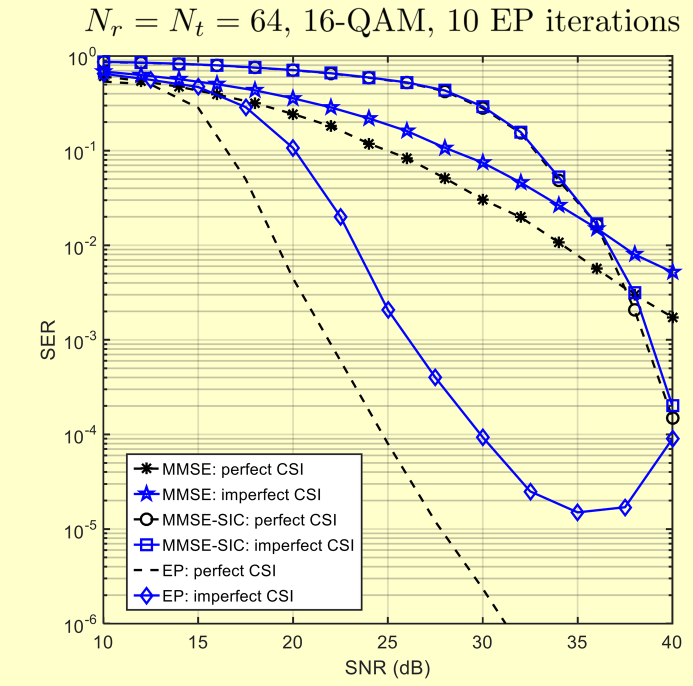
Constituent PDF: $p(\mathbf{u}|\mathbf{y}, H) \propto \mathcal{N}(\mathbf{y}|H\mathbf{u}, R_n)\prod_{i=1}^{2N_t}\mathbb{I}_{u_i\in\mathcal{A}_M}$
$\mathcal{A}_M = \Re(\tilde{\mathcal{A}}_M) \cup \Im(\tilde{\mathcal{A}}_M)$
Proposed approximation: $q(\mathbf{u}) \propto \mathcal{N}(\mathbf{y}|H\mathbf{u}, R_n)\prod_{i=1}^{2N_t}\underbrace{\mathcal{N}(u_i|m_i, \psi_i)}_{q_i(u_i)} \propto \mathcal{N}(\mathbf{u}|\mu, \Sigma)$

Refine $q(\mathbf{u})$ by EP: $q^*(\mathbf{u}) \propto \mathcal{N}(\mathbf{u}|\mu^*, \Sigma^*)$
Detect the i-th estimated symbol from $\mu^*(i)$ and $\mu^*(N_t + i)$.

$N_r = N_t = 32$, 16-QAM, 10 EP iterations



$N_r = N_t = 64$, 16-QAM, 10 EP iterations



EP Detection with Imperfect CSI

- With perfect CSI, receiver considers the received vector as $\tilde{\mathbf{y}} = \tilde{H}'\tilde{\mathbf{u}} + \tilde{\mathbf{n}}$.
- In practice, the perfect CSI is not available at the receiver
  – The channel estimation error
  – Outdated CSI
  – Quantization errors
- The available CSI at the receivre: $\tilde{H}$ ($\tilde{H} \neq \tilde{H}'$)
- The receiver's view of the received vector: $\tilde{\mathbf{y}} = \tilde{H}\tilde{\mathbf{u}} + \tilde{\mathbf{n}}$
- **The performance of the EP detector is adversely affected by low-quality CSI.**

Modified EP Detector

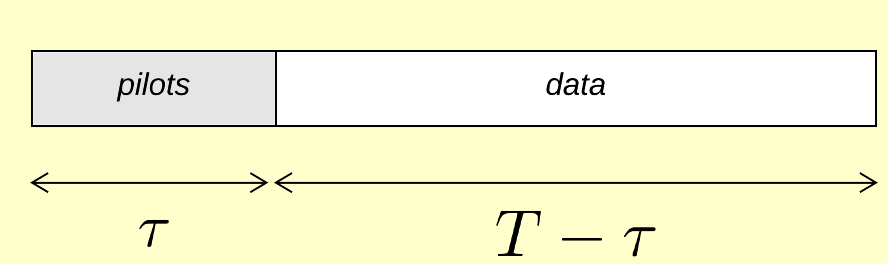
Let $\tilde{H} = \tilde{H}' - E$
E: the CSI error matrix
Assumption: E is caused due to **the channel estimation error**.

Modified receiver's model: $\tilde{\mathbf{y}} = \tilde{H}\tilde{\mathbf{u}} + \underbrace{E\tilde{\mathbf{u}} + \tilde{\mathbf{n}}}_{\tilde{\mathbf{w}}}$, $\mathbb{E}[\tilde{\mathbf{w}}] = \mathbf{0}$, and $\tilde{R}_w = E_s R_E + \sigma_n^2 I_{N_r}$

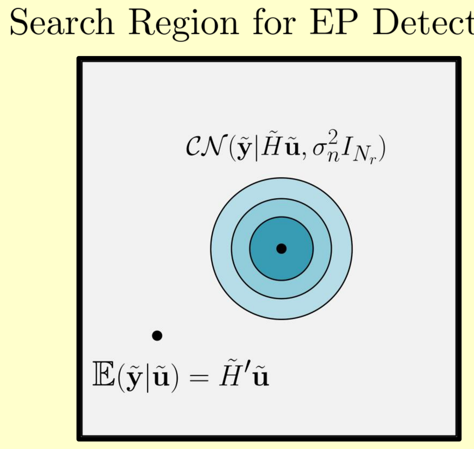
Calculating R_E:
The covariance matrix of $\mathbf{e} = \text{vec}(E)$ for MMSE channel estimator:
$R_e = \tilde{R}_h - \tilde{R}_h\tilde{P}^H(\tilde{P}\tilde{R}_h\tilde{P}^H + \sigma_n^2 I_{\tau N_r})^{-1}\tilde{P}\tilde{R}_h$

$$R_e = \begin{bmatrix} R_{1,1} & \ldots & R_{1,N_t}\\ \vdots & & \vdots\\ R_{N_t,1} & \ldots & R_{N_t,N_t}\end{bmatrix}$$

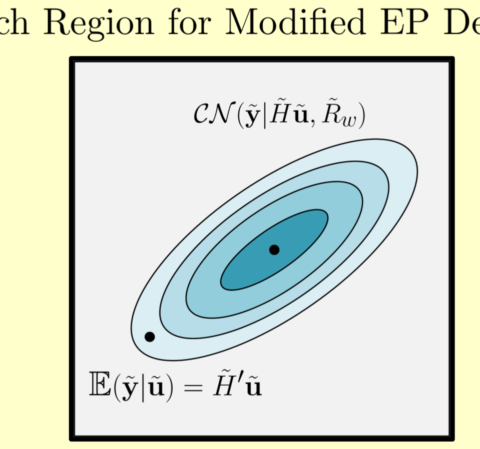
$R_E \triangleq \mathbb{E}[EE^H] = \sum_{i=1}^{N_t} R_{i,i}$



Search Region for EP Detector



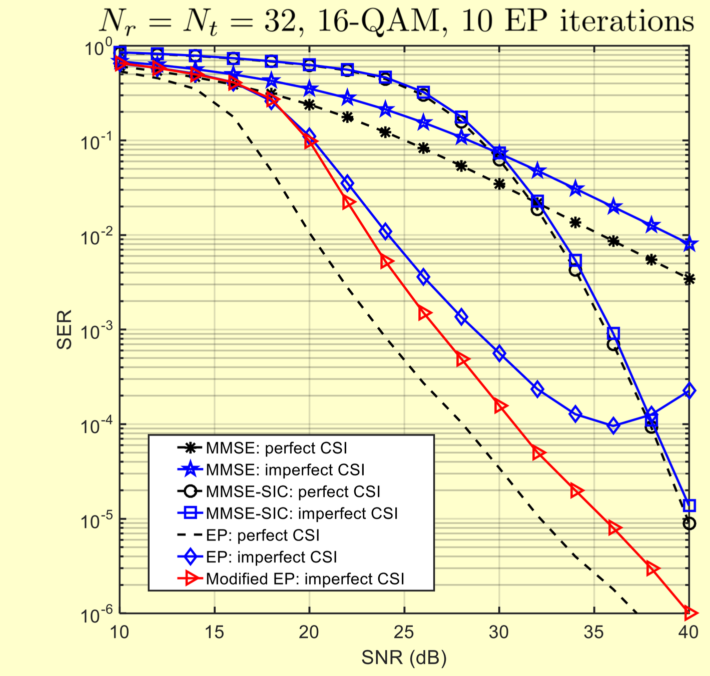
Search Region for Modified EP Detector



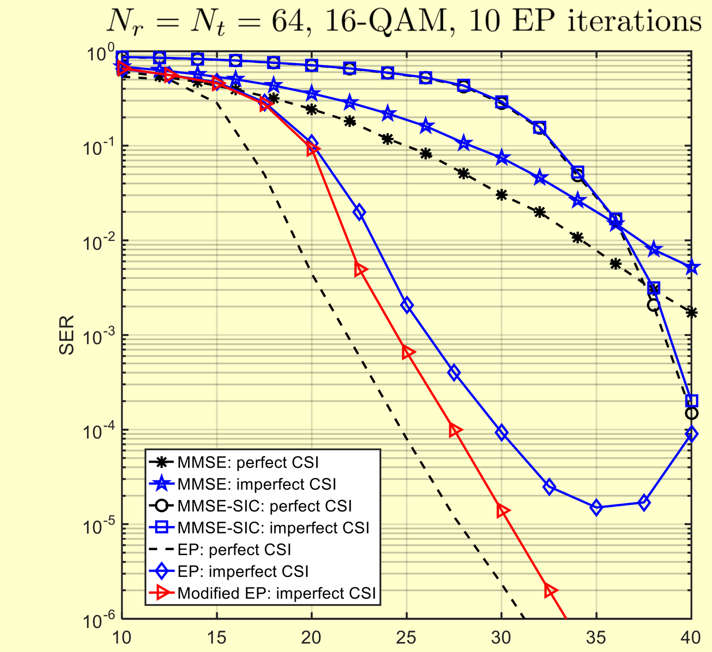
- $\tilde{H}'\tilde{\mathbf{u}}$: The actual conditional mean of $\tilde{\mathbf{y}}$.
- $\tilde{H}\tilde{\mathbf{u}}$: The encoder's view of the mean vector.
- **Low SNR**: The EP's search region covers the actual mean. However, the performance will be poor due to detrimental noise effects.
- **High SNR**: The EP's search region does not covers the actual mean. This adversly affects EP's convergence and degrades the SER performance.
- The effects of CSI error can be modeled with a colored noise.
- By considering the correlated noise model, the search region of the EP algorithm can be aligned toward the actual mean.

Simulation Results

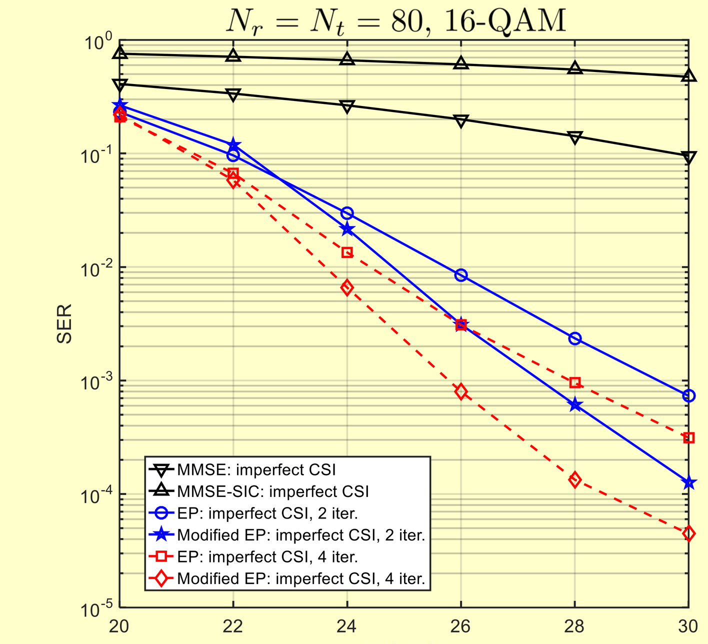
$N_r = N_t = 32$, 16-QAM, 10 EP iterations



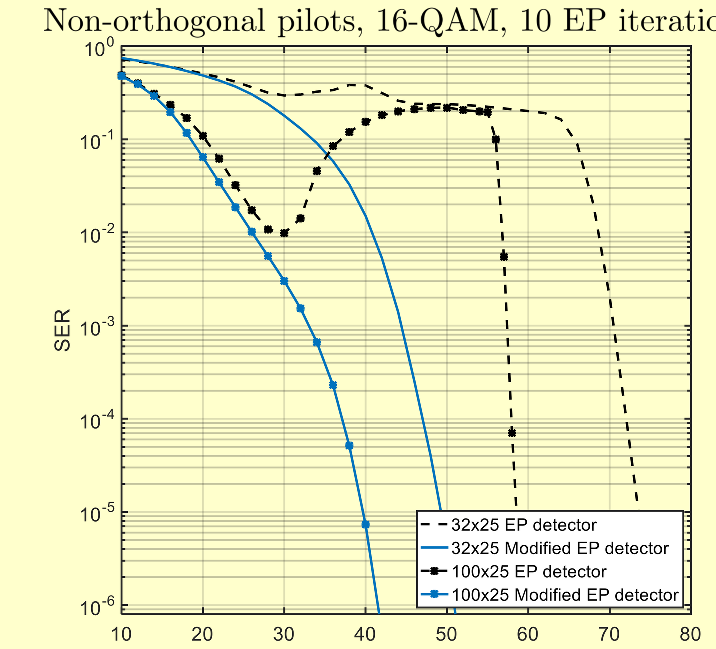
$N_r = N_t = 64$, 16-QAM, 10 EP iterations



$N_r = N_t = 80$, 16-QAM



Non-orthogonal pilots, 16-QAM, 10 EP iterations



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